Introducing the integrating factor $(x-y)^{-2}$, we have,

$$\frac{(x-y)(2xdx+2ydy)-(x^2+y^2)(dx-dy)}{(x-y)^2}=0,$$

which, upon integration, becomes

$$\frac{x^2+y^2}{x-y}=c, \qquad \text{or} \qquad x^2+y^2-c(x-y)=0.$$

Also solved by A. W. SMITH, NORMAN ANNING, J. W. CLAWSON, J. A. BULLARD, G. PAASWELL, O. S. ADAMS, ELIJAH SWIFT, FREDERICK WOOD, HORACE OLSON, C. A. BARNHART, L. M. COFFIN, G. W. HARTWELL, J. D. BOND, A. G. RAU, C. A. HUTCHINSON, CLARIBEL KENDALL, C. S. ATCHINSON, J. W. CROMWELL, C. P. SOUSLEY, J. A. CAPARO, and PAUL CAPRON.

405. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the greatest quadrilateral which can be formed with the four given sides a, b, c, and d taken in order.

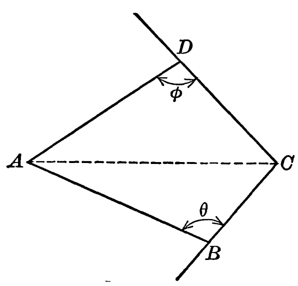
SOLUTION BY A. M. HARDING, University of Arkansas.

In the quadrilateral $ABCD$, if $AB=a$, $BC=b$, $CD=c$, $AD=d$, we have

$$\overline{AC}^2=a^2+b^2-2ab\cos\theta=c^2+d^2-2cd\cos\phi.$$

Hence,

$$ab\cos\theta-cd\cos\phi=\frac{a^2+b^2-c^2-d^2}{2}. \tag{1}$$



Also, area $=\frac{1}{2}ab\sin\theta+\frac{1}{2}cd\sin\varphi$. For a maximum or minimum we must have

$$ab\cos\theta d\theta+cd\cos\varphi d\varphi=0. \tag{2}$$

From (1) we obtain, by differentiation,

$$-ab\sin\theta d\theta+cd\sin\varphi d\varphi=0. \tag{3}$$

It follows from (2) and (3) that

$$\tan\phi=-\tan\theta; \qquad i.\,e., \qquad \phi+\theta=180°. \tag{4}$$

Therefore,

$$\cos\theta=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}.$$

Hence, the given quadrilateral will be a maximum when it can be inscribed in a circle.

Note.—It is evident from the nature of the problem that it is necessary to consider only *convex* quadrilaterals, and that (4) gives a maximum and not a minimum.

406. Proposed by C. N. SCHMALL, New York City.

Given $f(x+h)+f(x-h)=f(x)\cdot f(h)$, determine by Taylor's theorem or otherwise the nature of the function f.

SOLUTION BY W. M. CARRUTH, Hamilton College, New York.

Given

$$f(x+h)+f(x-h)=f(x)\cdot f(h). \tag{1}$$

Put $h=0$ in this equation. Then $2\cdot f(x)=f(0)\cdot f(x)$. Hence, either

$$f(x)=0, \tag{2}$$